



Substorms as nonequilibrium transitions of the magnetosphere

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Abstract

The complexity of the magnetospheric dynamics during substorms has attracted continued attention and its understanding requires approaches beyond the conventional one based on plasma processes. Recognizing the nonequilibrium and open nature of the coupled solar wind-magnetosphere system, attempts have been made recently to interpret the observational data in terms of self-organization (SO) and self-organized criticality (SOC). The SO concept reflects the nonlinear organized behavior of the magnetosphere as a whole, while the SOC approach emphasizes its multi-scale aspects. Evidence of both SO and SOC in substorm dynamics inferred from ground-based and multi-spacecraft data, as well as the possible combination of these concepts within the framework of a more general approach of nonequilibrium phase transitions are presented in this paper. Like the real sandpiles and recent SOC models, the multi-scale manifestations of the substorm activity are more consistent with the phase transition behavior, viz. a system effectively tuned to criticality rather than being self-organized. The problem of characterizing this critical behavior other than the widely used power-law frequency and scale spectra is discussed. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Substorms; Phase transition; Criticality

1. Introduction

The current understanding of the coupled solar wind-magnetosphere system has been based largely on the processes involving the plasma in the near-Earth geospace. The main endeavor in space physics thus has been to interpret and understand the extensive ground and spacecraft-based data in terms of the interactions between plasmas and electric and magnetic fields. However, the understanding of the complex magnetospheric dynamics apparent in the observational data of the dominant features such as the substorms has been a long-standing challenge. There are many reasons for the complex behavior of the magnetosphere. The inherent nonlinearity of plasma processes leads most linear processes such as instabilities to nonlinear states and the interaction among them in turn leads to the cross-scale coupling in the magnetosphere. The electrodynamic nature of the interaction between different parts of magnetosphere in the presence of the anchor magnetic field leads to a global coherence in its

dynamics. In the same time, a magnetosphere is an open system driven by the turbulent solar wind and consequently the dominant processes are essentially nonequilibrium in nature. These and other features of the magnetosphere indicate the limitations in the attempts to understand the complexity of magnetospheric substorms in terms of basic plasma processes alone.

The magnetospheric substorms are the most pronounced phenomena associated with the storage and release of the solar wind energy and momentum in the magnetosphere. They have typical time scales of hours, well-defined growth, expansion and recovery phases, and distinct signatures, viz. aurora brightening, sudden changes in the auroral electrojet (AE) indices, turbulence, current disruption, plasmoid formation and release in the magnetotail. In spite of the distributed nature of the physical processes and their apparent irregular behavior, there is a remarkable coherence in the magnetospheric response during substorms and the entire magnetosphere behaves as a global dynamical system (Baker et al., 1990; Siscoe, 1991) consistent with the low dimensionality obtained from time-series data (Vassiliadis et al., 1990). This approach has led to a number of studies based on the low-dimensional nonlinear dissipative

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behavior of the magnetosphere, which exhibits complex and irregular behavior due to phenomena such as dynamical chaos (Vassiliadis et al., 1990; Sharma, 1995). Along with the new understanding of magnetospheric dynamics, this has yielded many prediction tools with high efficiency and accuracy with direct application to space weather forecasting (Vassiliadis et al., 1995, 1996; Horton and Doxas, 1996; Klimas et al., 1996, 1997; Sharma, 1996). However, there are many features of the magnetosphere that need to be understood better beyond the low-dimensional behavior. Studies of the power spectrum of magnetic field fluctuations during the disruption of the geomagnetotail current (Ohtani et al., 1995; 1998) and the AE index (Tsurutani et al., 1990) have shown that there is a break in the spectrum and this has been interpreted in terms of bicolored noise (Takalo et al., 1993). These results have shown that while the fractal nature, and hence self-similarity and scale invariance, are clearly present, the dynamics may not be fully characterized as that of a low-dimensional system. This has motivated a view of the magnetosphere as a complex system and concepts such as self-organized criticality (SOC) has been introduced as a new way to understand its complexity and multi-scale nature (Chang, 1992; 1998; Consolini, 1997; Chapman et al., 1998; Uritsky and Pudovkin, 1998; Lui, 1998; Angelopoulos et al., 1999; Sitnov et al., 2000). These studies are based on the recent advances on the study of complexity, in terms of low-dimensionality (Abarbanel et al., 1993), SOC (Bak et al., 1987) and phase transitions (Gil and Sornette, 1996), and present a new approach to modeling the magnetospheric activity.

The dynamics of open and spatially extended systems, including sandpiles, are not described adequately by the conventional SOC models. In particular, real sandpiles may behave in a manner more reminiscent of a first-order phase transition (similar to the ordinary fold catastrophe (e.g., Gilmore, 1993)) than a second-order one (Nagel, 1992). In the case of substorms such as violations of the SOC behavior are present in the form of the statistics of chorus events observed on the ground and particle injections detected by spacecraft in the inner magnetosphere (Borovsky et al., 1993; Pritchard et al., 1996; Smith et al., 1996). It is shown in particular that the intensity and occurrence rate of substorms have a probability distribution with a well-defined mean — a feature that is at variance with SOC behavior. A promising way to reconcile the signatures of organized low-dimensional nonlinear behavior of the magnetosphere with those of scale invariance and SOC-like behavior is in terms of phase transitions (Sharma et al., 1993, 1999; Sitnov et al., 1998, 2000). Practically, all SOC models consider the magnetosphere to be an *autonomous* system. However, the magnetosphere is clearly an open system driven by the solar wind, and thus is *non-autonomous*. It is the large variability of the solar wind impinging the Earth that creates

qualitatively different manifestations of activity like storms, substorms, pseudo-breakups, convection bays, etc. Therefore, it is necessary to take solar wind and magnetosphere variables together to analyze magnetospheric dynamics. This can be accomplished using the time delay, embedding technique in which the reconstructed vector consists of the combined magnetospheric and solar wind data (Sharma, 1993; Sharma et al., 1993, 1999; Sitnov et al., 1998, 2000). According to this analysis, the magnetosphere exhibits signatures of low effective dimension and organized behavior in the form of first-order dynamical phase transition for the largest energy storage-release events. On the smaller scales, its behavior is scale-invariant consistent with SOC concept and the second-order phase transition.

In this paper we use the data from WIND, GEOTAIL and INTERBALL spacecraft to study the global and multi-scale aspects of substorm dynamics. This yields features that characterize substorms as nonequilibrium transitions and a comprehensive picture can be developed within the framework of nonequilibrium phase transitions.

2. Global substorm behavior from vB_s -AL data

The solar wind-magnetosphere data compiled by Bargatze et al. (1985) has been used extensively to study the input–output behavior during substorms. The data set consists of 34 intervals of correlated solar wind input and the magnetospheric response as an output. The solar wind input is the induced electric field vB_s , where B_s is the southward component of the interplanetary magnetic field (IMF) and v is the component of the solar wind velocity along the Earth–Sun axis. The magnetospheric output or response to the solar wind is represented by the auroral electrojet index AL.

The dynamics of a system can be reconstructed from the time-series data of a limited number of physical variables using time delay techniques (Abarbanel et al., 1993). Among these, the singular spectrum analysis (SSA) (Broomhead and King, 1986) is often used to obtain the main dynamical features inherent in the data. This technique has been used earlier to reconstruct the autonomous dynamics of the magnetosphere from the AE indices (Sharma et al., 1993). The singular spectrum analysis is based on the singular value decomposition (Press et al., 1992) and uses the properties of the trajectory matrix constructed from the time series data by time-delay embedding. For the magnetosphere represented by the vB_s -AL data the time delay vector at time t_i is defined as

$$X(t_i) = \{AL(t_i), \dots, AL(t_i - (m-1)\tau); \\ vB_s(t_i), \dots, vB_s(t_i - (m-1)\tau)\},$$

where τ is the time delay and m is the embedding dimension. The trajectory matrix Y is then a $N \times 2m$ matrix constructed from N values of $X(t_i)$ for $i=1, 2, \dots, N$. Since the averaging time for this data set is 2.5 min, the value of the embedding

dimension m is chosen to be 32 to provide a time window of 80 min, appropriate for substorms. The singular-value decomposition $Y = U W V^T$ provides the expansion of this matrix into a series of projections corresponding to different eigenvalues w_i of the $2m \times 2m$ covariance matrix $Y^T Y$. The first application of the singular spectrum analysis to dynamical systems (Broomhead and King, 1986) was based on the idea that the noise in the system will give rise to a noise floor in the spectrum of eigenvalues and the number of eigenvalues above the noise floor will yield an estimate of the effective dimension of the system. In fact, in many realistic cases, the singular spectrum has a clear power-law form with no sign of a noise floor, and in such cases, a limited number of leading eigenvalues and eigenfunctions may serve as a good approximation of the system, analogous to the so-called mean-field or Landau approximation (Landau, 1937). Such an approximation is often used in phase transition theory as a zero-level description but this often eliminates the features connected with either SOC or second-order phase transition.

The first principal component P_1 corresponding to the eigenvector V_1 is a measure of the solar wind input vB_s , averaged over the interval of about 80 min, i.e., $P_1 \propto -\langle vB_s \rangle$. The second principal component P_2 is similarly averaged AL index and $P_2 \propto \langle AL \rangle$. The third component P_3 reflects the difference between the nearly immediate (with the time delay about 20 min) value of the parameter vB_s and its earlier values (time delay around 1 h). Fig. 1 shows the global dynamics of the magnetosphere given by (P_1, P_2, P_3) obtained from the first 15 intervals of Bargatze et al. (1985) data set. In this figure, P_3 has been rescaled within the surface routine as $P_3 \rightarrow P_3 + 25$ when producing the regular grid on the plane $(P_3, -P_1)$. The set of point representing the trajectory of the magnetosphere in this 3D space chosen by SSA is then approximated by a 2D surface based on an additional assessment of the dimension of the system on the largest space and time scales (Sitnov et al., 2000). The surface plot is complemented by the plot of circulation flows, which reflect the evolution of the solar wind parameters P_1 and P_3 during the period under study. The growth phase of substorms is reflected in Fig. 1 by the upper (right) part of the surface, while the recovery phase corresponds to the lower (left) part. The ground state with $vB_s = AL = 0$ corresponds to the point $(P_3, -P_1) = (25, 0)$ while the substorm onset is located close to the median $P_3 = 25$ with the largest transitions being at the largest possible values of the parameter $-P_1$.

The reconstructed surface shown in Fig. 1 resembles the so-called temperature–pressure–density (TPD) diagram typical of equilibrium first-order phase transitions (e.g., Stanley, 1971). The dynamical or nonequilibrium transitions exhibit hysteresis phenomenon in which different values of output parameter (AL index) may correspond to the same set of input parameters (vB_s). The clear surface shown in Fig. 1 excludes the episodes in the original data set which exhibit hysteresis.

Both the TPD diagram and the corresponding circulation flows including the principal components P_1, P_2 and P_3 turn out to be surprisingly similar to the model of substorm as a cusp catastrophe proposed earlier by Lewis (1991). Also it is consistent with the low-dimensionality of the magnetosphere. However, the SSA of Bargatze et al. (1985) data reveals deviations from the ideal catastrophe picture. First, the dimension is not quite clear for smaller scales (less than $1/20$ of the largest scale) (Sitnov et al., 2000). Second, the singular spectrum itself does not show clearly the number of the principal components necessary to represent the dynamics to be 3 or even a larger number.

The above results, including the deviations from the ideal catastrophe picture, can be interpreted in terms of phase transitions. In fact, the catastrophe-like picture and multi-scale behavior are features of dynamical and nonequilibrium phase transitions (Hohenberg and Halperin, 1977; Gil and Sornette, 1996). The catastrophe picture is thus only one aspect of the whole phenomenon, which is associated with first-order dynamical phase transitions, while the deviations from the ideal catastrophe picture may be explained by the features of a second-order phase transitions near the critical point. Moreover, the first-order phase transition picture, which then plays a role of the mean-field approximation (Stanley, 1971), can yield the location of the critical point.

3. Multi-spacecraft data of the solar wind-magnetosphere system

Geomagnetic indices such as AL , AU , AE , and Dst are widely used in magnetospheric studies, including nonlinear dynamical studies and prediction. They have been derived continuously from the data from arrays of ground-based magnetometers following well-known procedures. However, the indices are known to have many limitations due to the modifications of the original data by averaging or taking the envelope of many different records. These modifications may introduce artificial features and cause the loss of essential dynamical information. They also complicate the task of relating the dynamical models based on these indices, the models based on the physical parameters and the studies of the fundamental processes. Another limitation of conventional indices is that they represent only the ground-based measurements of the basic processes, which take place deep in the magnetosphere, and thus are remote sensing data. With the availability of multi-spacecraft data of the magnetosphere, there is a unique opportunity to substitute indices by some more physical parameters directly related to basic mechanisms of geomagnetic activity.

A new multi-spacecraft database consisting of the solar wind parameters taken from WIND spacecraft and the magnetic field from the GEOTAIL and INTERBALL has been compiled (Sergeev, 1999; private communication). The solar wind data from WIND spacecraft have been

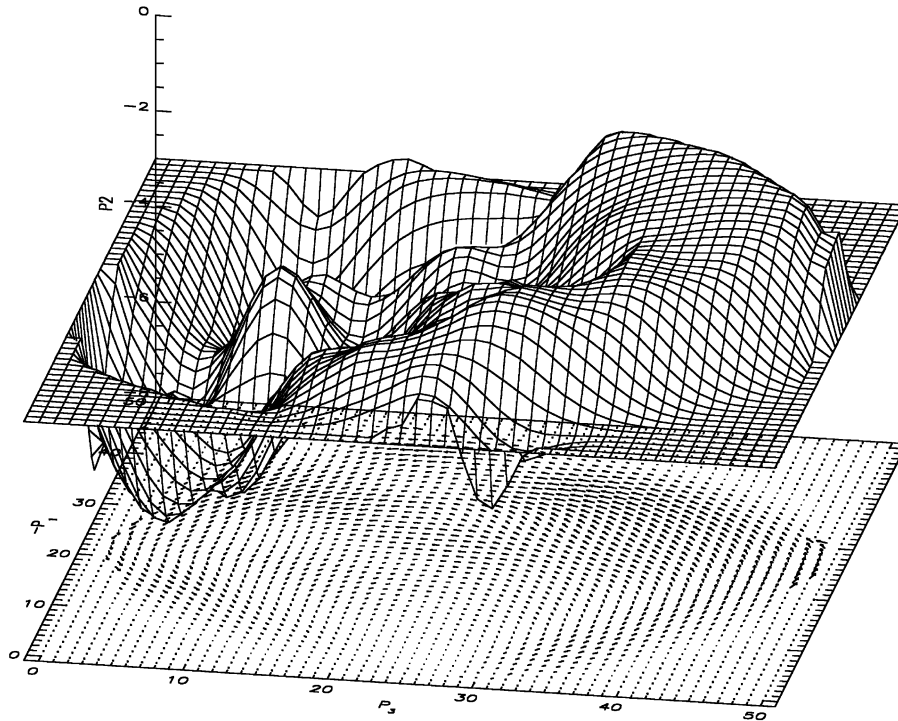


Fig. 1. The surface on which the trajectories of the state of the magnetosphere in 3D space (created by the eigenvectors corresponding to three largest SSA eigenvalues) have located. Data-derived circulation flows on the plane $(P_3, -P_1)$ reflect the evolution of the solar wind parameters during the substorm cycle.

selected to fit three basic criteria: (1) the spacecraft is close to the Sun–Earth line ($|Y, Z| < 15R_E$) to avoid large errors; (2) $X < 100R_E$ for the same reason; and (3) long enough records of the lobe field data from GEOTAIL and INTERBALL spacecraft are available in the selected intervals. The time resolution of the data set is 2 min.

The spacecraft part of the magnetospheric response is obtained from GEOTAIL and INTERBALL spacecraft. These spacecraft spend limited periods in the lobes and this poses the main limitation on the data base, which consists of seven intervals of nearly continuous measurements each around 2 or 3 days in length. To make the data less dependent of the spacecraft two procedures have been adopted. First, based on the results of Petrukovich et al. (1999) the effective lobe magnetic field was calculated using the simple pressure balance formula $(B_L)^2/8\pi = n(T_e + T_i) + B^2/8\pi$, which seems to be a good estimate of the lobe field even if the spacecraft is located in the plasma sheet. This procedure was used mainly for GEOTAIL data. Second, the measurements have been reduced to $20R_E$ along the tail axis using the statistical formula of Fairfield and Jones (1996). So far, seven data sets from the period December 1995 to December 1996 have been compiled using WIND, GEOTAIL and INTERBALL data, and the data for December 8, 1995 are shown in Fig. 2. One of the most striking features of this data set is the near

coincidence of the reduced lobe-field data for GEOTAIL and INTERBALL spacecraft. Thus the multi-spacecraft data provide us with independent evidence in favor of a considerable degree of coherence in the response of the magnetosphere to the solar wind changes.

4. Multi-scale behavior of the magnetosphere during substorms

The multi-scale behavior of the magnetosphere is evident from the singular spectra of the ground-based data (Sitnov et al., 2000). The eigenvalue spectrum for the Bargatze et al. (1985) data set is shown in the left panel of Fig. 3. For multi-spacecraft data described above the spectrum is shown in the right panel of Fig. 3. These plots show clear power-law dependence with the exponents themselves having values close to unity for both data sets, in agreement with the traditional interpretation of critical behavior (Jensen, 1998). Such critical behavior indicated by power spectra of different data sets was explained in terms of the SOC (Consolini, 1997; Chapman et al., 1998; Lui, 1998; Uritsky and Pudovkin, 1998; Chang, 1998; Angelopoulos et al., 1999). Based on a simplified mathematical model of a sandpile (Bak et al., 1987), SOC provides a seemingly universal mechanism of

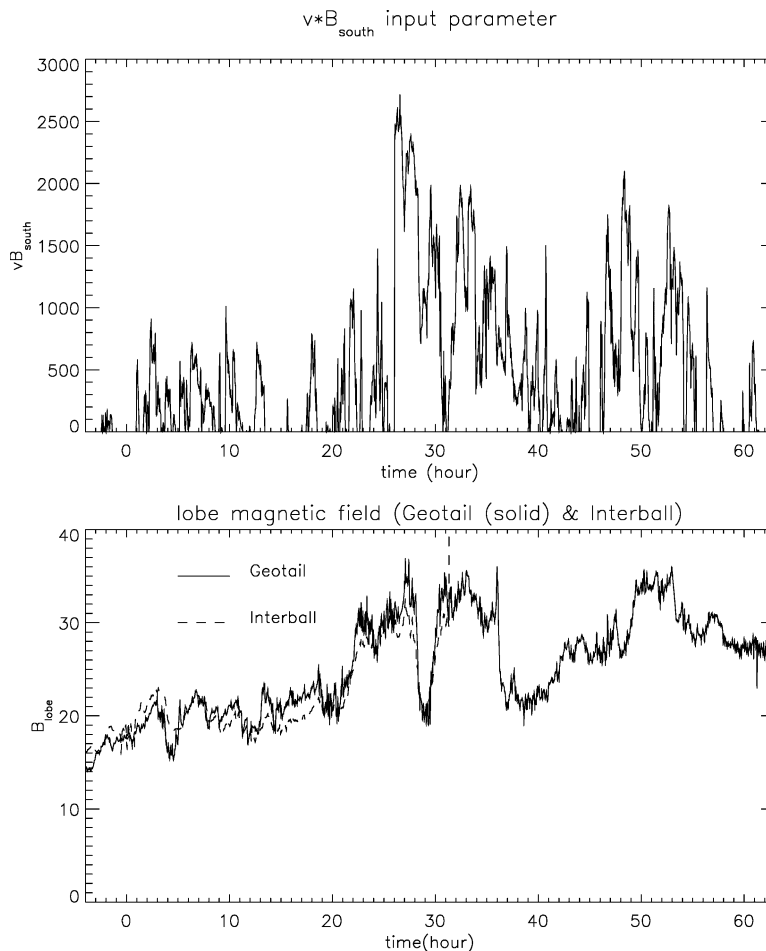


Fig. 2. The multi-spacecraft data of the solar wind vB_s from WIND and lobe magnetic field from GEOTAIL and INTEBALL. The global coherence of the magnetospheric response to solar wind energy input is evident from the near coincidence of the lobe magnetic field measurements by the two spacecraft.

persistent criticality, which requires no additional tuning of the system to keep it close to the critical point.

However, considering the dynamical behavior of real systems this is a simplified model and need to be modified. First, it has been found that the real sandpiles may behave in a manner more reminiscent of a first-order transition similar to the fold catastrophe than a second-order one (Nagel, 1992). In the case of substorm activity of the magnetosphere similar deviations from the simplest SOC picture are detected in the form of the statistics of chorus events seen at the ground and particle injections in the near-Earth magnetosphere (Borovsky et al., 1993; Pritchard et al., 1996; Smith et al., 1996), which revealed that the intensity and occurrence rate of substorms have a probability distribution with a well defined mean. Independent studies of SOC models themselves revealed that the critical points in some of them are not attractive. On the contrary, typical SOC models imply some specific tuning of either state (Gil and

Sornette, 1996) or control (Vespignani and Zapperi, 1998) parameters.

The way to improve the simplest SOC picture and reconcile it with the seemingly alternate SO interpretations is indicated by the fact that both SO and SOC aspects of complex system behavior demonstrate different aspects of *phase transition* behavior. Moreover, they seem to be organized in the same manner as conventional equilibrium phase transitions. The specific “temperature pressure — density” diagram of substorms described in Section 2 implies that the critical point must correspond to relatively small substorm activations. Similarly, only relatively small avalanches (from 3 to 80 grains) in real sandpile experiments (Jensen, 1998) demonstrate power-law behavior, while larger avalanches: on the contrary, demonstrate the distinctive features of the first-order transitions including the hysteresis phenomenon. An advanced SOC–SO model of the complex system consistent with the above sandpile experiments has been

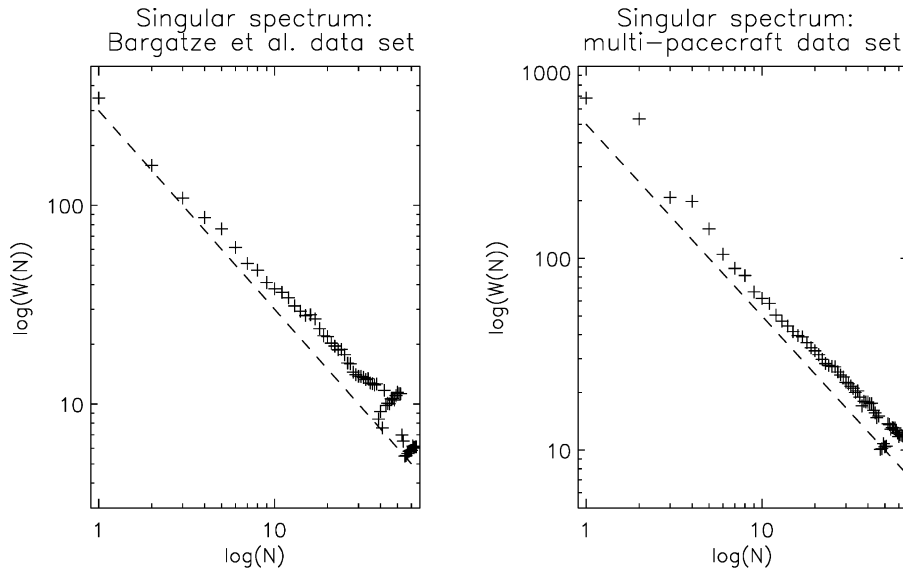


Fig. 3. Log-log SSA spectra for the first 15 intervals of Bargatze et al. (1985) data set and multi-spacecraft data set. Dashed lines correspond to the exponent equal to unity.

proposed recently by Gil and Sornette (1996) within the framework of Landau–Ginzburg theory of self-organized criticality. The SOC model proposed Chapman et al. (1998) to account for the global coherence of the dynamics is based on such a model. However, the conjecture that the global and multi-scale manifestations of the dynamics of complex systems may be organized in the same manner as equilibrium first- and second-order phase transitions, need to be studied further. The straightforward way to resolve this problem is to find an analogue of the power-law spectra discussed above, where the state parameter of the system would be related not to scale or frequency but to some input (control) parameter of the system as in actual second-order phase transitions or some advanced SOC models (Gil and Sornette, 1996). In other words we need to find the *genuine critical exponent* of the system. This is however, a complicated task due to the non-equilibrium nature of the magnetospheric transition. Contrary to equilibrium systems, where the critical exponent is determined by the form of the coexistence curve (Stanley, 1971), the nonequilibrium system may often reach the so-called spinodal curve corresponding, for instance, from overheated water to overcooled steam (e.g., Gunton et al., 1983). Moreover, due to the finite rate of the variation of the control parameters of the system, it passes the spinodal curve at finite rate, so that even this metastable state cannot be used effectively for determining an analogue of the critical exponent. It is expected that the analogue of the conventional critical component, which would describe some basic input–output properties of the magnetosphere can be found from the extensive data that are being obtained from multi-station ground-based and multi-spacecraft data.

5. Conclusions

Attempts to study and model the behavior of the magnetosphere on substorm and shorter scales have been developing along two main directions. The first direction is based on the low-dimensionality of the magnetosphere and its organized nonlinear behavior. It provides a consistent picture of a substorm as a sudden catastrophe-like transition of the magnetosphere with natural differentiation between spontaneous onsets (Dmitrieva and Sergeev, 1983; Henderson et al., 1996) and those triggered by the solar wind (Caan et al., 1975; Sergeev et al., 1996). It has resulted in very efficient prediction tools based on local linear autoregressive filters (Vassiliadis et al., 1995) and other low-dimensional models (Horton and Doxas, 1996; Klimas et al., 1997). Another direction with the emphasis on multi-scale aspect of the substorm dynamics is also consistent with other aspects of substorm dynamics (Sergeev et al., 1996) and allows some modeling based on the SOC concept (Chapman et al., 1998; Takalo et al., 1999). In the latter case predictions based on the statistical characteristics, e.g., in the spirit of earthquake forecasting based on the Gutenberg–Richter well-known law (Gutenberg and Richter, 1954) and of forest fires (Malamud et al., 1998) could be made.

In spite of some evidence (power-law spectra or effective dimension and TPD diagram) neither the SO nor SOC approach were taken separately explain the whole variety of the magnetospheric activity during substorms. However, the results presented in this and a recent paper (Sitnov et al., 2000) reveal the unifying physical picture, which combines these seemingly mutually incompatible concepts in a consistent manner analogous to the equilibrium phase

transition theory. As we have shown in this paper, the behavior of the Earth's magnetosphere is very much like with that of real sand piles (Jensen, 1998 and refs. therein). Both systems reveal scale-invariant behavior for relatively small avalanches and first-order phase transition-like behavior for largest avalanches. This suggests that the power-law spectra of the magnetospheric activity including that inferred from our SSA analysis might be the result of a dynamical second-order phase transition rather than the SOC manifestations, the conjecture which is confirmed now for some systems previously attributed as key SOC. It is believed that further development in this direction will result in a better understanding of the magnetosphere as a complex nonlinear system and a more efficient forecasting of its activity.

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